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Reconstruction of Abstract Quantum Theory

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Understanding quantum theory as a general theory of prediction, we reconstruct "abstract" quantum theory. "Abstract" means the general frame of quantum theory, without reference to a three-dimensional position space, to concepts like particle or field, or to special laws of dynamics. "Reconstruction" is the attempt to do this by formulating simple and plausible postulates on prediction in order to derive the basic concepts of quantum theory from them. Thereby no law of "classical" physics is presupposed which would then have to be "quantized." We briefly discuss the relationship of "theory" and "interpretation" in physics and the fundamental role of time as a basic concept for physics. Then a number of *assertions* are given, formulated as succinctly as possible in order to make them easily quotable and comparable. The assertions are arranged in four groups: heuristic principles, verbal definitions of some terms, three basic postulates, and consequences. The three postulates of separable alternatives, indeterminism, and kinematics are the central points of this work. These brief assertions are commented upon, and their relationship with the interpretation of quantum theory is discussed. Also given are an outlook on the further development into "concrete quantum theory" and some philosophical reflections.

1. THE PROBLEM

Quantum theory has been extremely successful. A set of axioms for quantum theory can be presented on one page of print; yet there may now well be 10^9 empirical facts obeying the theory and none contradicting it. In a series of papers (Weizsäcker, 1955, 1971, 1985a,b, 1986; Drieschner, 1970, 1979; Görnitz and Weizsäcker 1987a,b, 1988) we have tried to understand this success by interpreting quantum theory as a universal theory of prediction. This paper gives a brief, improved version of the interpretation.

By "abstract" quantum theory we designate the general frame of quantum theory, without reference to a three-dimensional position space,

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to concepts like particle or field, or to special laws of dynamics. "Reconstruction" means the attempt to formulate simple and plausible postulates on prediction and to derive the basic concepts of abstract quantum theory from them.

This enterprise entails a specific methodological problem. We want to reconstruct a basic theory of physics. In physics, the term "theory" means a mathematical structure together with a physical, preferably empirical semantics. We may assume the mathematical structure to be well-defined; but without semantics (without rules of application) it is not yet physics. The semantics, however, must be expressed verbally in an available language. Thus, the semantics shares the ambiguity in the meaning of words that prevails in everyday language. In fact, we only learn a more precise meaning of those words in the long run of applying them within the empirical use of the theory; Einstein said "only the theory tells us what can be observed" (cf. Heisenberg, 1969). In formulating our present postulates, we cannot avoid using the present-day language of physics, as it has developed after 60 years of unchallenged application of quantum theory. We desire, however, to reduce this language as far as possible to a meaning independent of the present state of consciousness of the scientific community. We try to use terms that are as "phenomenological" as possible, describing phenomena that are elements of everybody's experience.

A short description of abstract quantum theory as it is used today may introduce the reader to our actual enterprise. We are probably not mistaken if we say that abstract quantum theory comprises only four basic concepts:

1. Hilbert space as a state space.
2. Probability relations between states as defined by the Hilbert metric.
3. Composition of objects by the tensor product of their state spaces.
4. Dynamics as a unitary state space representation of the additive real group of time translations.

Each of these four phrases expresses a combination of a mathematical structure with a term of physical semantics. "Hilbert space," "metric," "tensor product," "unitary group representation" are four purely mathematical terms. "State," "probability," "object," "composition," "time," "dynamics" are six terms of physics. Without this second class of terms, the theory is not physics at all. The meaning of these terms is, however, not self-evident. Every attempt at further explaining them is in general called an "interpretation" of quantum theory (Görnitz and Weizsäcker, 1987a).

Our reconstruction differs from most traditional descriptions or interpretations of quantum theory in one central point. It does not presuppose any set of laws of "classical" physics which would then have to be "quantized." This statement holds in two different levels of abstraction. On the lower level: We do not presuppose any concepts such as position space, particle, or field, hence neither classical particle mechanics nor field theories. We introduce quantum particles or fields only as a *consequence* of abstract quantum theory under the title of a "concrete quantum theory"; this program is briefly indicated in Section 5 and shall be developed in later papers. On the higher level of abstraction: We do not even presuppose a general frame of classical laws, such as a Hamiltonian formalism. We expect that such a formalism can be deduced from abstract quantum theory by a process of going to a limit; but this is not a condition we would impose on our postulates. Our procedure is closest to what is traditionally called quantum axiomatics. Our early papers started from some concept of quantum logic (Weizsäcker, 1955; Drieschner, 1970).

The level of abstraction on which we propose to start is indicated by the six "terms of physics" contained in our description of abstract quantum theory. "Object" (also called "system" in physics), "state," and "composition of objects" are terms of some "abstract ontology of physics"; in our own procedure we even replace "object" by the purely logical concept of "statement." "Time" is the only intuitive concept without which, in our view, even the most abstract set of laws cannot be physics. "Dynamics" means the laws under which states change in time. "Probability" we consider as the prediction of relative frequency, thus as an abstract concept referring to the future, hence to time.

We proceed in four steps. In Section 2 we express a number of *assertions*. We have tried to formulate them as briefly as possible, in order to make them easily quotable and comparable. But their very brevity makes them in need of *comment*, which is given in Section 3; thus, it might be helpful to read a group of assertions together with its commentary. The relationship with what is usually called the *interpretation* of quantum theory is discussed in Section 4. We end in Section 5 with an outlook on the further development into "concrete quantum theory" and with some philosophical reflections.

The assertions of Section 2 are arranged in four groups:

A. Heuristic principles
B. Verbal definitions of some terms
C. Three basic postulates
D. Consequences

The heuristic principles describe our methodology. The definitions cannot, at the outset, do more than narrow down the aura of ambiguity around every word in our language. The three postulates try to express the essence of quantum theory in the language prepared by the definitions. The consequences aim at deducing from the postulates precisely the structure described by the four basic concepts of the traditional abstract quantum theory; yet we will see that the "deduction" is not yet logically strict, but needs a few additional "assumptions of simplicity."

2. ASSERTIONS

A. Heuristic Principles

A1. Preconditions of Experience

As far as possible, our postulates ought to express conditions without which we cannot expect experience to be possible at all.

A2. Simplicity

Without precisely defining simplicity, we wish for simple postulates rather than complicated ones.

A3. Innocous Generality

General rules are usually simpler than specialized ones. We shall confine ourselves to general rules as far as they give the hope of being "innocuous"; e.g., claiming the general existence of a set of states, while under special conditions (such as a dynamics implying a superselection-rule) some of those might not actually come into being.

B. Definitions

B1. Experience

Experience means to learn from the past for the future.

B2. Facticity of the Past

We speak of past events as of objective facts, independently of our actually knowing them.

B3. Possibility of the Future

We are aware of future events only as possibilities.

B4. Probability

Probability is a quantification of possibility. We define it as the prediction (mathematically: the expectation value) of a relative frequency.

B5. Temporal Statements

A temporal statement (briefly "statement") is a verbal proposition (or a mathematical proposition with a physical meaning) referring to a moment in time.

B6. States

States are recognizable events. A state is what is the case when some temporal statement is true. States at different times can be identical: it is meaningful to ask whether we observe now the same states as at a certain time before.

B7. Conditional Probability

Let x and y to be two states. Then $p(x, y)$ is the probability that, if x is a present state, y will be found as the state if searched for.

B8. Alternatives

An n-fold alternative is a set of n mutually exclusive states, exactly one of which will turn out to be present if and when an empirical test of this alternative is made.

B9. Connection

Two states x and y are called connected if there is a law of nature determining their conditional probabilities $p(x, y)$ and $p(y, x)$. If the connection is transitive, i.e., if the existence (by law of nature) of probabilities $p(x, y)$ and $p(y, z)$ implies the existence of a $p(x, z)$, then connection is an equivalence relation, defining a partition of the class of all states into subclasses of mutually connected states.

B10. Separability

Two states are called separable if they are not connected.

C. Postulates

C1. Separable Alternatives

There are alternatives whose states are separable from nearly all other states. "Nearly" will be defined as meaning all states not connected with the states of the alternative by postulate C2.

C2. Indeterminism

If x and y are two connected, mutually exclusive states $\{p(x, y) = p(y, x) = 0\}$, there are states z that are not logically constructed from x and y by mere logical operations and which possess conditional probabilities $p(z, x)$ and $p(z, y)$ none of which is equal to zero or to one.

C3. Kinematics

The conditional probabilities between connected states are not altered when the states change in time: $p[x, t), (z, t)] = p[(x, 0), (z, 0)]$.

D. Consequences

D1. State Space

We call the set of states connected with a separable alternative its state space. With innocuous generality we assume the state spaces of all separable n-fold alternatives A_n to be isomorphic: $S(n)$.

A state $z \in S(n)$ defines n conditional probabilities $p(z, x_i)$, where x_i $(i = 1, \ldots, n)$ are the states defining the n-fold alternative; $\sum_{i=1}^{n} p(z, x_i) = 1$.

D2. Completeness

For any mathematically possible set of values $p(z, x_i)$ there is a state z in $S(n)$. We assume this to be an example of innocuous generality.

D3. Equivalence of States

All states in $S(n)$ are equivalent. Else their distinction would be an additional alternative connected with A_n. Hence, A_n would not have been separable.

D4. Symmetry Group

The equivalence of the elements of $S(n)$ is expressed by a symmetry group $G(n)$ which preserves the conditional probabilities between them. Due to D3, $G(n)$ must be a continuous group.

D5. Alternatives in S(n)

Due to D3, there exists a $p(x, y)$ between any two states x and y of $S(n)$. The equivalence of all states in $S(n)$ further implies that any $z \in S(n)$ is a member of a precisely n-fold alternative of mutually exclusive states of $S(n)$.

D6. Metric in S(n)

As an "assumption of simplicity," we suppose $G(n)$ to be a simple Lie group. There are two simple Lie groups preserving a relation of mutual exclusion between precisely n normalized vectors by preserving a metric: $O(n)$ and $U(n)$. Thus, we assume $S(n)$ to permit a faithful irreducible representation in an n-dimensional vector space $V(n)$, $G(n)$ being either orthogonal or unitary.[3] The states of $S(n)$ will then correspond to normalized vectors in $V(n)$, i.e., to one-dimensional subspaces.

D7. Dynamics

According to C3, the change of state in time must be a one-parameter subgroup $D(t)$ of $G(n)$. We call the special choice of such a subgroup the choice of a law of dynamics.

D8. Preservation of State

If a state is to be recognizable in time, there must exist a possible law of dynamics which keeps this state constant.

D9. Complexity

The generator of $D(t)$, as defined in D7, must, according to D8, permit diagonalization. This is universally possible only if V is complex, and, due to the metric, a Hilbert space. Hence $G(n) = U(n)$.

D10. Composition

Two alternatives A_m and A_n are simultaneously decided by deciding their Cartesian product $A_{m \cdot n} = A_m \times A_n$. The $A_{m \cdot n}$ defines the Hilbert space $V(m \cdot n) = V(m) \otimes V(n)$.

3. COMMENTARY

A1. Preconditions of Experience

Here we translate an ancient philosophical idea into a merely heuristic principle. Plato and Aristotle knew already that universal laws cannot be

[3]Here we only consider linear representations, which are always possible. We have not studied the consequences of nonlinear representations, which might lead beyond traditional quantum theory. Furthermore, $O(n)$ preserves a linear metric and $U(n)$ a sesquilinear one. Linear metrics can be constructed on vector spaces over the real and complex numbers and over the quaternions; sesquilinear metrics exist in spaces over the complex numbers and the quaternions. Because the quaternions can be represented by 2×2 matrices of complex numbers and because there are no groups for the quaternions that are different from the groups over real or complex vector spaces, we guess that there is no need to construct a quantum mechanics over quaternions.

logically deduced from an always incomplete selection of empirical findings. Hume emphasized that universal laws are supposed to hold in future experience, which is not yet available when we formulate the laws. It was Kant's idea that universal laws will necessarily hold in experience only if they express no more than those conditions without which experience would not be possible at all. He considered logic and mathematics, space and time, and concepts such as substance and causality as such preconditions of scientific experience. His system has not withstood the impact of modern physics. Yet we shall examine our definitions and postulates by asking the extent to which they may be considered as "epistemic," i.e., as necessary conditions of empirical knowledge.

A2–A3. Simplicity and Generality

The idea of "innocuous generality" expresses no more than the methodological statement that it is possible to form correct general laws without referring to the special cases that fall under them, with the meaningful explanations of exceptions. Thus, "mammals have four legs" holds true even if some animal has lost one leg by an accident and hence has empirically only three legs.

B1–B3. Time and Experience

Experience is gained in time. Hence the structure of time is a precondition of experience. Experience has been gained from past facts; it is used to predict the possibilities of the future. The philosophy of time is not the subject matter of this paper. We want to state that we do not consider the description of past and future in B2–B3 as "only subjective." But our further argument makes no use of this view. If we confine ourselves to describing quantum theory as a theory on human knowledge, then these two descriptions are certainly to be presupposed. Hence, the expressions: "we speak of...," "we are aware of...."

B4. Probability

In traditional quantum theory, probability is a central concept, introduced by the statistical interpretation, in which it refers to prediction, hence to the future, i.e., to time. Our definition contains, so we believe, the correct elements of the three competing interpretations of probability: the *empirical* one (relative frequency), the *subjective* one (a prediction always contains the uncertainty, yet not of arbitrary opinion, but of the unknown future), and the *logical* one (probability of any statement: prediction of the expected outcome of a test).

B5. Temporal Statements

These obey special logical rules (Weizsäcker, 1985, Chapter 2). We propose to renounce the use of the truth values "true" and "false" for statements on the future, replacing them by modalities like "possible," "necessary," and their negations. Probability is then a quantification of these modalities.

B6. States

"State" is another central term in traditional quantum theory. Our definition uses the undefined term "event," which explicitly refers to time, and the fact that events can be subsumed under descriptive concepts, logically speaking under universals. "The event of the rising sun."

B7–B8. Conditional Probability and Alternatives

These definitions are not as harmless as they may look. They contain the dependence of quantum-theoretic predictions on observation in the phrases "if searched for" and "test of this alternative." In classical physics, these two specifications would be unnecessary. Yet the specifications are not begging the question, since they are more modest than their classical omission. Our formulation would still permit the classical assumption that the specifications are superfluous. Quantum theory is not actually introduced before our postulate C2. We confine ourselves in this paper to finite alternatives. Countably infinite alternatives will be introduced in a paper on "concrete quantum theory" by the definition of a particle. This means that in a higher approximation an infinite number of finite alternatives are connected as "possible properties of one object." The simplest well-known example is the representation of the Hilbert space of a free particle by the sum of the finite spaces belonging to the possible values of its angular momentum. A similar procedure is repeated in quantum field theory, combining an infinite set of separable Hilbert spaces of possible single particles in one inseparable space.

B9–B10. Connection and Separability

In classical physics one would say that connected states are states of the same object. We begin by the logical concept of an alternative and aim at reconstructing the concept of an object. This will not be fully achieved by abstract quantum theory. The ensuing comment on C1, separable alternatives, shows that the concept of approximate separability and hence of separable objects presupposes a concept of position space for its semantic consistency; thus, it presupposes "concrete quantum theory." The logical position of the assumption that connection is transitive seems to be as follows: It is only relevant if there are separable states. Then it implies the existence of classes of mutual connection such as used in C1, C2, D2. Conversely, these three assumptions imply the transitivity of connection.

C1. Existence of Separable Alternatives

It would seem that this assumption is a precondition of rational experience, i.e., of experience in which we can test the use of concepts by testing assumed laws of nature. A testable law ought at least to predict probabilities, and a testable probability is a conditional probability: "if x, then in the average y in the fraction $p(x, y)$ of performed tests." Yet quantum theory itself implies that C1 is not strictly true. In the composition of alternatives (and hence of objects) all states that are not products of states of the components make the probabilities of one state space dependent on the choice of the alternative decided in the other one (EPR). If we refer EPR to "nonlocality" (Cramer, 1986), then this structure applies not only to location in position space, but to any decidable alternative. We can call this statement the essential "holism" of quantum theory. Thus, one postulate C1 defines only the limited human approach toward the analysis of the wholeness of reality. The real question is: Why is C1 in most known cases so good an approximation? More precisely: Why are there such cases at all? Since, if there are cases in which C1 applies with good approximation, then it is not surprising that finite beings such as humans, especially in a rationalistic civilization like ours, should have preferred to study these cases. Our tentative answer to the question is: because cosmic space is nearly empty, hence locally separated objects are easily found. Why this, in turn, should be so is a question for another paper.

C2. Indeterminism

This is the name given to the postulate in the paper of Drieschner (1970). In fully developed quantum theory the postulate turns out to be equivalent to the principle of superposition. In the book of Weizsäcker (1985) it was called the postulate of extension. It was not meant as expressing some metaphysical truth but as a phenomenological description of the open future. But we cannot consider it as simply "epistemic" in the sense of our comment on A1. It seems indeed to imply a non-Boolean algebra of possible events and thus to be the decisive rejection of classical physics. The philosophical analysis of this step goes beyond the scope of the present paper. We introduce it here just as the simplest single postulate for quantum theory that we have been able to find. In the formulation of the postulate we explicitly reject states (or statements) that can be constructed from the given x and y by logical operations, i.e., by "and" and "or." Negation is in fact used in the definition of mutual exclusivity, but it is not employed for the definition of states like z. This is only a methodological decision. Quantum axiomatics is usually described by postulates that refer to the lattice of "properties" of an object; the papers of Drieschner (1970, 1979) work in this lattice, too. Weizsäcker (1985, Chapter 8.3), chooses our present approach, which, in the lattice language, means confining the theory to the "atoms" of the lattice. The motive was the tendency to clearly separate the problem of indeterminism, which refers to the maximal possible knowledge of states, from the propositional lattice, which, except for the atoms, expresses incomplete knowledge. It would be the task of another paper fully to translate the consideration of Drieschner (1979) into the present "atomic" language.

We presuppose that $p(x, y) = 0$ implies $p(y, x) = 0$. This assumption may be called the mutuality of exclusion. It follows from the law of double negation, $\neg\neg a = a$. Intuitionism has rejected this law for infinite sets, and we might doubt whether we can accept it in temporal logic for the open future. We consider it, however, as justified for finite alternatives and —a problem only to be discussed in "concrete quantum theory"—for infinite-dimensional but separable state spaces.

C3. Kinematics

We tend to consider this postulate as epistemic and call it the "Darwinism of states." How can states connected with a separable alternative be identified if their only mutual relations are not preserved through time? But this it not a strict argument; we offer it as a plausibility.

D1, D2, and D4. Symmetry of the State Space

The traditional problem of quantum axiomatics is how to arrive at linearity, i.e., at a vector space. Drieschner (1970) followed Jauch (1968) and others, introducing a lattice of events, proceeding toward a projective geometry and hence to the embedding vector space. Our present approach (Weizsäcker, 1985) confines itself to "pure states" and introduces the vector space as a representation space of their symmetry group. The symmetry is not just postulated, but implied by separability. Since separability is only an approximation, we cannot maintain that abstract quantum theory, founded on it, should be a final theory; it may itself be an approximation at an even more "holistic" theory.

D3. Equivalence of States

This can be argued in two steps. First, the states x_i that define the alternative are mutually equivalent, since their distinction would be a connected alternative. Second, all states of $S(n)$ are equivalent, since the definition of the x_i as members of an alternative refers to observation from outside, hence to interaction with other alternatives. As long as $S(n)$ is strictly separated, it will not be determined whether a given state $z \in S(n)$ is an element of an alternative that would be decided by introducing outside interaction.

From this second step it follows that there exists a $p(x, y)$ between any two states x and y of $S(n)$.

D5–D6. Metric

We have not studied the general theory of continuous groups that would preserve a relation of "mutual exclusion" of precisely n elements of a set. Hence, within the limits of our present mathematical insight, we must admit the possibility of an essentially different, more complicated mathematical structure fulfilling our postulates. The empirical success of quantum mechanics favors our choice. At present, we can imagine three alternative explanations of this success:

a. There are more "epistemic" postulates excluding the other possibilities.
b. All epistemically meaningful more complicated structures can be mathematically decomposed into several applications of our structure.
c. There is a more general theory, which has not yet come to the mind of physicists.

We remark that the argument as presented in D5–D6 differs from the less precise consideration of Weizsäcker (1985, Chapter 8.3).

D7 and D9. Dynamics and Complexity

It is a traditional problem why quantum theory, if represented in a vector space, should use a complex space. Pauli (1932), in his answer to Ehrenfest (1932), starts out from a second-order differential equation in time (reversibility) and from the postulate that there should be a probability density for position. We introduce reversibility by C3 and D6, i.e., by postulating dynamics to be described by a group. We replace Pauli's probability postulate by the definition of conditional probabilities B7, and by postulating that it should be mathematically possible to define a law of dynamics that would preserve the state B6, D7.

D8. State Preservation

This postulate was not used in Weizsäcker (1985). There, a real vector space was first introduced, and dynamics was defined so as to preserve its real metric as given by the conditional probabilities. Under such a dynamical law a $2n$-dimensional real space would admit a description as an n-dimensional complex space. But the new complex metric would then differ from the original real one, and it would depend on the choice of the dynamical law.

Our present assumption of state preservation might be qualified as another "assumption of simplicity." Why should it be impossible to measure time by the steadily moving hands of a clock? Yet what we assume is only that there should be a "possible" law of dynamics making the hands stand still. And it is to be admitted that in a real vector space of even dimension a one-dimensional orthogonal group will not keep any state constant at all. Thus the choice between $O(n, R)$ and $U(n)$ seems to be definitely in favor of the latter.

D10. Composition

The generalized EPR-nonlocality (comment on C1) turns out to be a consequence of postulate C2.

4. INTERPRETATION

Our reconstruction has a double aim. It is intended (1) as an amplification of the Copenhagen Interpretation (CI), and (2) as a first step toward a reconstruction of "concrete quantum theory."

We consider the Copenhagen Interpretation not as one of several possible interpretations of a self-consistent theory called "quantum mechanics," but as the attempt at giving that minimal semantics to the formalism of quantum mechanics without which one would not know how to apply the formalism to reality at all, i.e., without which it would not yet be a theory in the sense of physics. Precisely for this reason it has never been possible to "codify" CI, since, as pointed out in Section 1, semantics shares the ambiguities of everyday language. Semantics develops with the application of the theory.

We compare our approach with the early CI in three respects: (1) correspondence with classical physics, (2) statistical interpretation of the wave function, and (3) the role of the observer.

4.1 Correspondence

Heisenberg's quantum mechanics was a mathematical model of the theory at which Bohr's correspondence principle had aimed. Observable quantities such as position, momentum, and energy were known from classical physics, but now obeyed different mathematical laws, which, for large quantum numbers, implied classical mechanics as a limiting case. Bohr insisted later that also in quantum mechanics observations ought to be described in classical terms. In this historical setting, both classical and quantum theories are justified by their empirical success, connected with mathematical simplicity.

Within the realm of physics, our approach is not one of correspondence. We begin by abstract quantum theory without any empirical or classical specification of observables; this specification we reserve for the ensuing step of concrete quantum theory. However, we still keep the essence of Bohr's insistence on classical terms, though transferred from the field of physics into the field of logic. Bohr maintained that only classical terms describe the phenomena in an unambiguous manner. The corresponding, though far more restrained statement in our approach is the postulate C1, craving the approximate existence of separable alternatives.

4.2. Probability

Like CI we accept the statistical nature of quantum theory as given. The amplification lies in our analysis of time as a precondition of experience. Probability is the only available scientific description of the open future. This, in our description, is not an absolute, metaphysical statement, but a phenomenological one. This is what we know today. Determinism for separable alternatives is excluded only by the holistic consequences of postulate C2. The difference between fact and possibility is consistent with the theory of irreversibility (see (Weizsäcker, 1939; 1985, Chapter 4).

4.3. The Observer

The misunderstandings and hence the criticisms of CI were mainly connected with the role of the observer. The "collapse of the wave function by observation" is the crucial expression; quantum mechanics seems to describe "knowledge instead of reality." In our view, most of the dissatisfaction derives from an unwieldy phraseology. The wave function is the catalogue of those probabilities that are mathematically implied by the knowledge gained in an experiment. Probabilities in physics are conditional and change by the awareness of new conditions. "Knowledge" means to *know* a *reality.* The difficulty arises from a neglect of the difference between past and future. Past facts can be known; possibilities for the future are guesswork, guided by probabilities, which can be empirically tested as expectation values of relative frequencies in an ensemble.

This is what we know today.

5. OUTLOOK

The outlook is twofold: (1) concrete quantum theory, a theory we hope to present; (2) philosophy, an unending task.

5.1. Concrete Quantum Theory

Traditional quantum theory accepts the concepts of time, space, particle, and field, hence of motion, position, momentum, energy, and force from classical physics. In our reconstruction we only use time from the outset, and we replace all concepts of objects by the logical concept of alternative, all concepts of temporal properties of objects by the concept of state. Time, however, we describe in a more detailed manner than classical physics; while we also measure it by a real variable t, we make explicit use of its "modes"—present, past, future—with their qualitative differences. The resulting "abstract" theory is general enough to serve as a *frame* for introducing all the above-mentioned traditional "concrete" terms.

Yet, in ensuing papers, we hope to demonstrate that all these concepts, including the theories referring to them, such as relativity and particle theory, can be developed as a *consequence* of abstract quantum theory. This is essentially to be done by one single (and simple) idea: the reduction of all alternatives to the successive decision of *binary alternatives* (yes–no decisions, bits or "urs").

The binary ($n=2$) alternative defines a C^2 state space with $SU(2)$ symmetry. Systems defined by a Cartesian product of binary alternatives possess a state space which is, or is a subspace of, the tensor product $C^2 \otimes C^2 \otimes \cdots$. Objects with this state space will have $SU(2)$ as a symmetry of their dynamics. $SU(2)$ is locally isomorphic to $SO(3)$, and we start from the working hypothesis that this is the reason for a three-dimensional real space offering a natural description of all objects in physics: the "position space." Relativity, and particles as irreducible representations of a relativistic group, can be derived from this hypothesis (Scheibe *et al.*, 1958; Weizsäcker, 1971, Chapter 115; 1985, Chapters 9 and 10; Castell, 1975; Drieschner, 1979; Görnitz 1986, 1987).

This is an unfinished program. If completed, it might indeed demonstrate that all known basic theories of physics are consequences of abstract quantum theory.

5.2. Philosophy

Learning from Socrates and Bohr, we might define philosophy as the pursuit of semantics, as the unending quest: "Do we know what we mean by what we say?" Bohr sometimes said: "What is the difference between science and other mental enterprises? In science, at least, we do not from the outset give up any hope that in the end our concepts might be just a little bit clearer than in the beginning." Philosophy of science then would be the attempt to become aware of this clarification, as far as it can be presently achieved.

In this paper we end by discussing the three terms "continuity," "consciousness," and "holism."

5.2.1. Continuity

In our reconstruction, the mathematical continuum is only used as measure of time, and as the set of possible values for probability. In "concrete quantum theory" space also will be measured by probabilities, defined by the group parameters of the basic Lie group $SU(2)$. We cannot exclude the possibility that time, as an observable of a final quantum theory, would equally be ultimately measured by probabilities.

Now we remember that quantum theory was historically made necessary by the "ultraviolet catastrophe," i.e., by the impossibility of thermodynamic equilibrium in the actual infinity of degrees of freedom in a classical mechanical continuum. Thus, quantum theory seems to replace the idea of an actually existing physical continuum throughout by the continuum of probabilities, i.e., of possibilities.

This reminds us of the view which was traditional from Aristotle to the time of Gauss, that infinity means no more than indefinite possibility (of counting, of dividing, etc.). Cantor defended his idea that the natural numbers form an actually infinite set (from which he proceeded to higher cardinalities) against the objections of the Aristotelians by the remark: "If you can actually count on indefinitely, the set of *possible* numbers is actually infinite," i.e., he, too, defended infinity (and, consequently, continuity) as an infinity of possibilities. We might hence think of quantum theory precisely as the adequate theory of physical continuity.

5.2.2. Consciousness

One of the nonexisting "paradoxes" of CI is the idea that the reduction ("collapse") of the wave function by an observation means the sudden nonlocal change of an "objective" wave under the causal influence of an immaterial agency called the consciousness of the observer. In Section 4 we have already discussed the sufficient answer: The wave function is no more than a catalogue of conditional probabilities, i.e., predictions following from present knowledge. The "paradox" lies only in the "holism" to which we shall presently return. But what is the role of "consciousness" which is implied by the term "knowledge"?

Consciousness or knowledge may appear in a double role in natural science. The first role is inescapable: science is knowledge. The observer (the "knower") is introduced explicitly into the interpretation of quantum theory only because of its probabilistic structure: the probabilities depend essentially on the knowledge of the present state, hence this knowledge must be explicitly mentioned in order to define the prediction catalogue as presently valid. Here the knower is the conscious *subject* posessing the knowledge.

Completely different is the hypothetical second role of consciousness, the role as known, as the *object* of knowledge. Bohr explicitly excluded consciousness as a possible object of quantum theory. He was even skeptical whether quantum theory might be able to describe the phenomena of organic life. He did not try to answer questions beyond the scope of physics as known to him by the futile attempt to apply this physics to them.

In this respect the present authors dare to go beyond Bohr. The Cartesian distinction between mind and matter is not a necessary part of an abstract quantum theory, which can in principle be applied to any empirically decidable alternative, e.g., "where is this electron?" or "what will I feel two minutes from now?" Quantum theory would not exclude even a philosophy of "spiritualistic monism" in which virtual consciousness would be an attribute of all reality, and the brain might just be the classical limit, the physically (i.e., externally) observable "surface" of the mind. This remark is neither present science nor strict philosophy; it is rather an attempt to indicate an open frontier.

5.2.3. Holism

The older word "holism" (see, e.g., Smuts 1927; Meyer-Abich, 1948) has recently been taken up again for describing quantum theory, with good reason. The "realism" of classical physics believed in the existence of separable objects, only connected by "interaction." In the quantum theory of a compound object, only a set of measure zero describes separated (product) states of its parts. Hence, conceptual thinking which refers to separable objects is necessarily only an approximation. If we permit ourselves to leave behind also the separation of "mind" and "matter," the adequate philosophy of quantum theory would seem to resemble Platonism or Vedanta, in which the basic reality is called neither matter nor spirit, and certainly not a set of separate substances, but the One. Again, this statement is not a theory, it is an open frontier.

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